Trigonometrija - osnovne formule

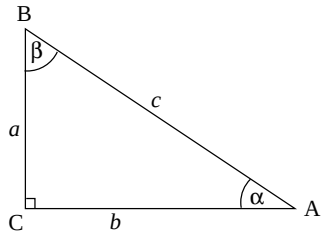
Definicija trigonometrijskih funkcija – u pravokutnom trokutu:

$$\sin\alpha = \frac{\text{nasuprotna kateta}}{\text{hipotenuza}} = \frac{a}{c}$$

$$\cos\alpha = \frac{\text{priležeća kateta}}{\text{hipotenuza}} = \frac{b}{c}$$

$$\operatorname{tg}\alpha = \frac{\text{nasuprotna kateta}}{\text{priležeća kateta}} = \frac{a}{b}$$

$$\operatorname{ctg}\alpha = \frac{\text{priležeća kateta}}{\text{nasuprotna kateta}} = \frac{b}{a}$$

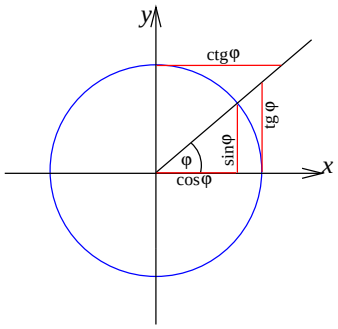


Vrijednosti trigonometrijskih funkcija u nekim važnijim kutevima:

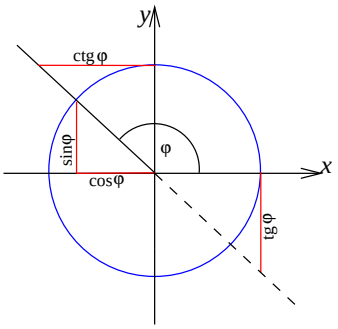
stupnjevi:	$0°$	$30°$	$45°$	$60°$	$90°$	$120°$	$150°$	$180°$	$270°$	$360°$
radijani:	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2}{3}\pi$	$\frac{5}{6}\pi$	π	$1\frac{1}{2}\pi$	2π
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	0	-1	0
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	-1	0	1
tg	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	$\pm\infty$	$-\sqrt{3}$	$-\frac{\sqrt{3}}{3}$	0	$\pm\infty$	0
ctg	$\pm\infty$	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0	$-\frac{\sqrt{3}}{3}$	$-\sqrt{3}$	$\mp\infty$	0	$\mp\infty$

Vrijednosti trigonometrijskih funkcija na jediničnoj kružnici:

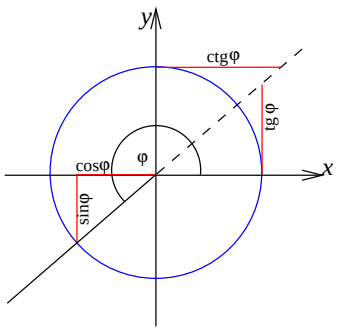
Prvi kvadrant:
$\sin\varphi > 0$
$\cos\varphi > 0$
$\operatorname{tg}\varphi > 0$
$\operatorname{ctg}\varphi > 0$



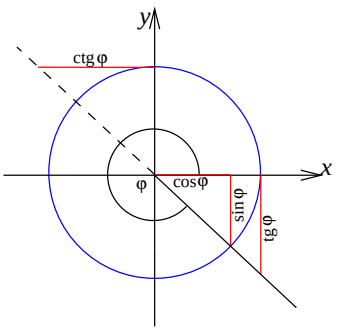
Drugi kvadrant:
$\sin\varphi > 0$
$\cos\varphi < 0$
$\operatorname{tg}\varphi < 0$
$\operatorname{ctg}\varphi < 0$



Treći kvadrant:
$\sin\varphi < 0$
$\cos\varphi < 0$
$\operatorname{tg}\varphi > 0$
$\operatorname{ctg}\varphi > 0$



Četvrti kvadrant:
$\sin\varphi < 0$
$\cos\varphi > 0$
$\operatorname{tg}\varphi < 0$
$\operatorname{ctg}\varphi < 0$



Osnovne trigonometrijske formule:

$$\sin^2\alpha + \cos^2\alpha = 1$$

$$\operatorname{tg}\alpha = \frac{\sin\alpha}{\cos\alpha}$$

$$\operatorname{ctg}\alpha = \frac{\cos\alpha}{\sin\alpha}$$

- **Parnost i neparnost trigonometrijskih funkcija:**

$$\sin(-\alpha) = -\sin\alpha \qquad \cos(-\alpha) = \cos\alpha \qquad \operatorname{tg}(-\alpha) = -\operatorname{tg}\alpha \qquad \operatorname{ctg}(-\alpha) = -\operatorname{ctg}\alpha$$

- **Izražavanje jedne trigonometrijske funkcije pomoću druge:**

$$\sin\alpha = \sqrt{1-\cos^2\alpha} = \frac{\operatorname{tg}\alpha}{\sqrt{1+\operatorname{tg}^2\alpha}} = \frac{1}{\sqrt{1+\operatorname{ctg}^2\alpha}} \qquad \cos\alpha = \sqrt{1-\sin^2\alpha} = \frac{1}{\sqrt{1+\operatorname{tg}^2\alpha}} = \frac{\operatorname{ctg}\alpha}{\sqrt{1+\operatorname{ctg}^2\alpha}}$$

$$\operatorname{tg}\alpha = \frac{\sin\alpha}{\sqrt{1-\sin^2\alpha}} = \frac{\sqrt{1-\cos^2\alpha}}{\cos\alpha} = \frac{1}{\operatorname{ctg}\alpha} \qquad \operatorname{ctg}\alpha = \frac{\sqrt{1-\sin^2\alpha}}{\sin\alpha} = \frac{\cos\alpha}{\sqrt{1-\cos^2\alpha}} = \frac{1}{\operatorname{tg}\alpha}$$

- **Funkcije zbroja i razlike kutova:**

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta \qquad \cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

$$\operatorname{tg}(\alpha \pm \beta) = \frac{\operatorname{tg}\alpha \pm \operatorname{tg}\beta}{1 \mp \operatorname{tg}\alpha \cdot \operatorname{tg}\beta} \qquad \operatorname{ctg}(\alpha \pm \beta) = \frac{\operatorname{ctg}\alpha \cdot \operatorname{ctg}\beta \mp 1}{\operatorname{ctg}\beta \pm \operatorname{ctg}\alpha}$$

- **Funkcije dvostrukih kutova:** $\sin 2\alpha = 2\sin\alpha \cdot \cos\alpha$ $\qquad \cos 2\alpha = \cos^2\alpha - \sin^2\alpha$

$$\operatorname{tg}2\alpha = \frac{2\operatorname{tg}\alpha}{1-\operatorname{tg}^2\alpha} \qquad \operatorname{ctg}2\alpha = \frac{\operatorname{ctg}^2\alpha - 1}{2\operatorname{ctg}\alpha}$$

- **Funkcije polovičnog kuta:** $\sin\dfrac{\alpha}{2} = \sqrt{\dfrac{1-\cos\alpha}{2}}$ $\qquad \cos\dfrac{\alpha}{2} = \sqrt{\dfrac{1+\cos\alpha}{2}}$

$$\operatorname{tg}\frac{\alpha}{2} = \sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}} = \frac{1-\cos\alpha}{\sin\alpha} = \frac{\sin\alpha}{1+\cos\alpha} \qquad \operatorname{ctg}\frac{\alpha}{2} = \sqrt{\frac{1+\cos\alpha}{1-\cos\alpha}} = \frac{1+\cos\alpha}{\sin\alpha} = \frac{\sin\alpha}{1-\cos\alpha}$$

- **Zbroj i razlika trigonometrijskih funkcija:**

$$\sin\alpha + \sin\beta = 2 \cdot \sin\frac{\alpha+\beta}{2} \cdot \cos\frac{\alpha-\beta}{2} \qquad \sin\alpha - \sin\beta = 2 \cdot \cos\frac{\alpha+\beta}{2} \cdot \sin\frac{\alpha-\beta}{2}$$

$$\cos\alpha + \cos\beta = 2 \cdot \cos\frac{\alpha+\beta}{2} \cdot \cos\frac{\alpha-\beta}{2} \qquad \cos\alpha - \cos\beta = -2 \cdot \sin\frac{\alpha+\beta}{2} \cdot \sin\frac{\alpha-\beta}{2}$$

$$\operatorname{tg}\alpha \pm \operatorname{tg}\beta = \frac{\sin(\alpha \pm \beta)}{\cos\alpha \cdot \cos\beta} \qquad \operatorname{ctg}\alpha \pm \operatorname{tg}\beta = \pm\frac{\sin(\alpha \pm \beta)}{\sin\alpha \cdot \sin\beta}$$

$$\operatorname{tg}\alpha + \operatorname{ctg}\beta = \pm\frac{\cos(\alpha - \beta)}{\cos\alpha \cdot \sin\beta} \qquad \operatorname{ctg}\alpha - \operatorname{tg}\beta = \pm\frac{\cos(\alpha \pm \beta)}{\sin\alpha \cdot \cos\beta}$$

- **Produkt funkcija:**

$$\sin\alpha\sin\beta = \frac{1}{2}\left[\cos(\alpha-\beta) - \cos(\alpha+\beta)\right] \qquad \cos\alpha\cos\beta = \frac{1}{2}\left[\cos(\alpha-\beta) + \cos(\alpha+\beta)\right]$$

$$\sin\alpha\cos\beta = \frac{1}{2}\left[\sin(\alpha-\beta) + \sin(\alpha+\beta)\right]$$

- **Rješavanje trokuta:**

- Sinusov poučak: $\dfrac{a}{\sin\alpha} = \dfrac{b}{\sin\beta} = \dfrac{c}{\sin\gamma}$ ili $a : b : c = \sin\alpha : \sin\beta : \sin\gamma$

- Kosinusov poučak: $a^2 = b^2 + c^2 - 2bc \cdot \cos\alpha$ $\qquad b^2 = a^2 + c^2 - 2ac \cdot \cos\beta$ $\qquad c^2 = a^2 + b^2 - 2ab \cdot \cos\gamma$

$$\cos\alpha = \frac{b^2+c^2-a^2}{2bc} \qquad \cos\beta = \frac{a^2+c^2-b^2}{2ac} \qquad \cos\gamma = \frac{a^2+b^2-c^2}{2ab}$$

- **Jednostavne trigonometrijske jednadžbe:**

$\sin x = a$	$x = (-1)^k \sin^{-1} a + k\pi$
$\cos x = a$	$x = \pm\cos^{-1} a + 2k\pi$
$\operatorname{tg}\, x = a$	$x = \operatorname{tg}^{-1} a + k\pi$
$\operatorname{ctg}\, x = a$	$x = \operatorname{ctg}^{-1} a + k\pi$

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